Université de Sherbrooke

33e rencontre sur la théorie des représentations des algèbres et sujets connexes

Résumés/abstracts

From (derived) Hall algebras to acyclic quantum cluster algebras.
Xueqing Chen

Inspired by Caldero-Keller's discovery of the similarity between the multiplication formulas in a cluster algebra and that in a (dual) Hall algebra, we firstly discuss an algebra homomorphism from the dual Hall algebra associated to $\mathrm{Rep}(Q)$ (category of representations of an acyclic quiver Q) to the corresponding quantum cluster algebra. Then we address the connection from two certain quotients of subalgebras of the derived Hall algebras of $\mathrm{Rep}(Q)$ to acyclic quantum cluster algebra. Finally, we give cluster multiplication formulas via the above derived Hall algebras. This talk is based on the joint works with Ming Ding and Fan Xu, and with Ming Ding and Haicheng Zhang respectively.

Superunitary regions, generalized associahedra, and friezes of Dynkin type cluster algebras.
Emily Gunawan

Each Dynkin diagram corresponds to a finite type cluster algebra $\mathbf{A}$, a commutative ring with distinguished generators called cluster variables which are grouped into overlapping sets called clusters. "Finite type" means that there are finitely many cluster variables, and these cluster variables are parametrized by the positive roots and negative simple roots of the corresponding root system. Each pair of clusters is connected by a sequence of involutions called mutations. These involutions can be visualized with the exchange graph, which has a vertex for each cluster and an edge for each mutation. The exchange graph of $\mathbf{A}$ is the 1-skeleton of a polytope called the generalized associahedron (for type A this is an associahedron, and for type B or C this is a cyclohedron). The faces of the generalized associahedron are indexed by the subclusters (subsets of clusters), and the facets are indexed by the cluster variables.

In this talk, we introduce the superunitary region of $\mathbf{A}$ which is the space of ring homomorphisms from $\mathbf{A}$ to the real numbers which send each cluster variable to at least 1. This region can be embedded into the positive orthant of the Euclidean space by requiring all cluster variables (which can be written as positive Laurent polynomials) to be bigger or equal to 1. Our main result is that the superunitary region of $\mathbf{A}$ is a regular CW complex which is homeomorphic to the generalized associahedron of $\mathbf{A}$. An application of this is a uniform proof that there are finitely many Dynkin-type positive integral friezes (functions on repetition quivers, classical combinatorial objects which have been studied since the 70s by Conway and Coxeter).This is joint work with Greg Muller.

Bethe algebras for $\mathfrak{gl}(n)$, Gelfand-Tsetlin patterns, and cacti.
Iva Halacheva

The Bethe subalgebras of the Yangian $Y(\mathfrak{gl}(n))$ are a family of maximal commutative subalgebras parametrized by diagonal invertible matrices with pairwise distinct eigenvalues. This family can be extended to the Deligne-Mumford compactification $M(0, n+2)$. If we fix a tame representation V of $Y(\mathfrak{gl}(n))$, for any point in the real locus of $M(0, n+2)$ the corresponding Bethe subalgebra acts on V with simple spectrum, leading to a covering of the real locus with fiber the eigenlines in V. We study the monodromy action for this covering, which can be realized combinatorially as a type of cactus group action on Gelfand-Tsetlin keystone patterns. This is joint work with Anfisa Gurenkova and Lenya Rybnikov.

Rooted labeled trees and exceptional sequences of type B_n/C_n.
Kiyoshi Igusa

Buan, Marsh and Vatne (2010) showed that clusters in the cluster category of type B_n or C_n are in bijection with maximal rigid objects in the cluster tube of rank $n+1$. Igusa-Todorov (2017) showed that ordered clusters in the cluster category are in bijection with signed exceptional sequences in the module category. For $n = 3$, the module category of type B_n has $n^n = 27$ exceptional sequences and the abelian tube of rank $n+1 = 4$ has $(n+1)^n = 64$ exceptional sequences. But, both categories have the same number of signed exceptional sequences ($(2n)!/n! = 120$). In fact we show there is a bijection between signed exceptional sequences of any length k in the abelian tube and in the module category of type B_n (combining the bijections of [BMV] and [IT]). We use combinatorial models: rooted labeled trees from my previous work with Emre Sen and pointed chord diagrams.

Remark: This is a joint work with Emre Sen.

Quasi-universal representations and generic bricks.
Colin Ingalls

This is joint work with Emily Cliff and Charles Paquette. Given a quiver, a dimension vector and a stability condition, Alistair King shows that, if the dimension vector is unimodular, there is a moduli space of stable representations with a universal representation. We extend this result to show that even when the dimension vector is divisible, the moduli space constructed by King has a quasi-universal representation. We are able to use the quasi-universal representation to construct generic bricks in several cases.

Truncated polynomials and Hamiltonian systems.
Michael Lau

Toda systems were introduced in the late 1960s as integrable lattice models for nonlinear particle interactions. Their Hamiltonians can be reinterpreted in terms of root systems, with coadjoint orbits serving as classical phase spaces. We will explain how these models can be extended to larger symplectic manifolds using Lie algebras of matrices over rings of truncated polynomials. We will then discuss exact solutions of the associated equations of motion, and how to lift the related classical integrable systems to families of commuting operators in universal enveloping algebras.

The (virtual) cactus group.
Joel Kamnitzer

I will introduce the cactus group, a relative of the braid group. Like its more famous cousin, it appears naturally in representation theory and is the fundamental group of a very natural space. I will describe this space, called the moduli space of genus 0 real curves. Then I will discuss a variant of the cactus group, which we call the virtual cactus group. Along the way, we will see the solution to the following question: what is the space of solutions to the equation $a + b = c$, where a, b, c are elements of $\mathbb{RP}^1$?

On Gorenstein algebras of finite Cohen-Macaulay type.
Ralf Schiffler

This is a report on joint work with Khrystyna Serhiyenko. We study the (stable) category of Cohen-Macaulay modules over 2-Calabi-Yau tilted algebras, a class of non-commutative algebras given by a quiver with potential. We are particularly interested in the case where the CM category is finite. We show the following.
(a) For a particularly nice subclass, which we call dimer tree algebras, the stable CM-category is a 2-cluster category of Dynkin type A.
(d) Every dimer tree algebra gives rise to several skew-group algebras, and for each of these the stable CM-category is a 2-cluster category of Dynkin type D.
(e) We have examples of Dynkin types E.
The dimer tree algebras are characterized by two conditions on the quiver. For one, we want that every arrow lies in an oriented (chordless) cycle, and moreover, the dual graph of the quiver is a tree. For dimer tree algebras, we obtain a combinatorial model for the CM category in terms of 2-diagonals in a regular polygon. For the type D, we have a similar model on a punctured polygon.

Higher Auslander algebras and Dynkin quivers.
Emre Sen

In the derived category of mod-KQ for Dynkin quiver Q, we construct a full subcategory in a canonical way, so that its endomorphism algebra is a higher Auslander algebra of global dimension $3k + 2$ for any $k \geq 1$. Furthermore, we extend this construction for higher analogues of representation finite algebras. Specifically, if M is an n-cluster tilting object in the bounded derived category of n-representation finite algebra, then we construct a full subcategory in a canonical way, so that its endomorphism algebra is a higher Auslander algebra of global dimension $(n + 2)k + n + 1$ for any $k \geq 1$.
As an application, we revisit the higher Auslander correspondence. Firstly, we describe the corresponding module categories that have higher cluster-tilting objects, and then we discuss their relationship with certain full subcategories of the derived category. Consequently, resulting algebras can be realized as endomorphism algebras of certain subcategories of (higher) cluster categories.

Centralizers of products of $\mathcal{L}U_q(\mathfrak{sl}_2)$-modules at roots of unity.
Charles Sénécal

Let V be the fundamental representation of the quantum group $U_q(\mathfrak{sl}_2)$. Quantum Schur-Weyl duality says that the centralizer of the action of $U_q(\mathfrak{sl}_2)$ on the product $V^{\otimes n}$ is isomorphic to the Temperley-Lieb algebra $\mathsf{TL}_n(q + q^{-1})$, even when q is a root of unity (in which case we consider the action of Lusztig's extension $\mathcal{L}U_q(\mathfrak{sl}_2)$).

We explore products other than $V^{\otimes n}$, namely we describe the centralizer of the action of $\mathcal{L}U_q(\mathfrak{sl}_2)$ on $P \otimes V^{\otimes n}$ and $L \otimes V^{\otimes n}$, where P and L are respectively a projective and a simple $\mathcal{L}U_q(\mathfrak{sl}_2)$-module. In particular, we give a description of the algebra $\mathrm{End}_{\mathcal{L}U_q(\mathfrak{sl}_2)}(P \otimes V^{\otimes n})$ in both cases when q is a root of unity or not. We also present an action of the blob algebra on the space $L \otimes V^{\otimes n}$ and determine its kernel. This is joint work with Yvan Saint-Aubin.

Generalized Kauer moves in Brauer graph algebras.
Valentine Soto

Brauer graph algebras are finite-dimensional algebras that are constructed from a graph called a Brauer graph. Kauer has proved that one can obtain a derived equivalence between Brauer graph algebras from the move of one edge in the corresponding Brauer graphs. Moreover, this derived equivalence can be interpreted in terms of silting mutations. I will explain how this result can be generalized to the move of multiple edges.

Universal F-polynomials and the u-equations.
Hugh Thomas

There is a system of non-linear equations associated to any finite dimensional algebra of finite representation type (the "u-equations"). The non-negative solutions to these systems of equations encode the structure of the tau-tilting fan and are relevant to the physics of scattering amplitudes. I will discuss an approach to deriving these equations from another, arguably more natural system of equations, in terms of what we call universal F-polynomials. For Dynkin quivers, the universal F-polynomials are a variant of the usual F-polynomials from cluster algebra theory, using universal coefficients instead of principal coefficients. Our perspective on u-equations was developed in the hereditary case by Arkani-Hamed, He, and Lam. This project can be viewed as part of an effort to extend some cluster algebraic phenomena to non-hereditary finite representation type algebras. This project is joint work with Nima Arkani-Hamed, Hadleigh Frost, Pierre-Guy Plamondon, and Giulio Salvatori.

2-roots for simply laced Weyl groups.
Tianyuan Xu

We introduce and study "2-roots", which are symmetrized tensor products of orthogonal roots of Kac–Moody algebras. We concentrate on the case where W is the Weyl group of a simply laced Y-shaped Dynkin diagram with three branches of arbitrary finite lengths a, b and c; special cases of this include types D_n, E_n (for arbitrary $n \geq 6$), and affine E_6, E_7 and E_8.

We construct a natural codimension-1 submodule M of the symmetric square of the reflection representation of W, as well as a canonical basis $\mathcal{B}$ of M that consists of 2-roots. We prove that with respect to $\mathcal{B}$, every element of W is represented by a column sign-coherent matrix in the sense of cluster algebras. We also prove that if W is not of affine type, then the module M is completely reducible in characteristic zero and each of its nontrivial direct summands is spanned by a W-orbit of 2-roots. Finally, we define and describe highest 2-roots, which are analogous to highest roots of usual root systems. (This is joint work with Richard Green.)